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SESSION ID: CRYP-M01A

The Window Heuristic: Automating Differential Trail Search in ARX Ciphers with Partial Linearization Trade-offs

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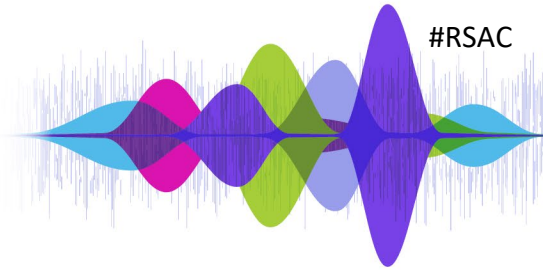

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Automated Cryptanalysis

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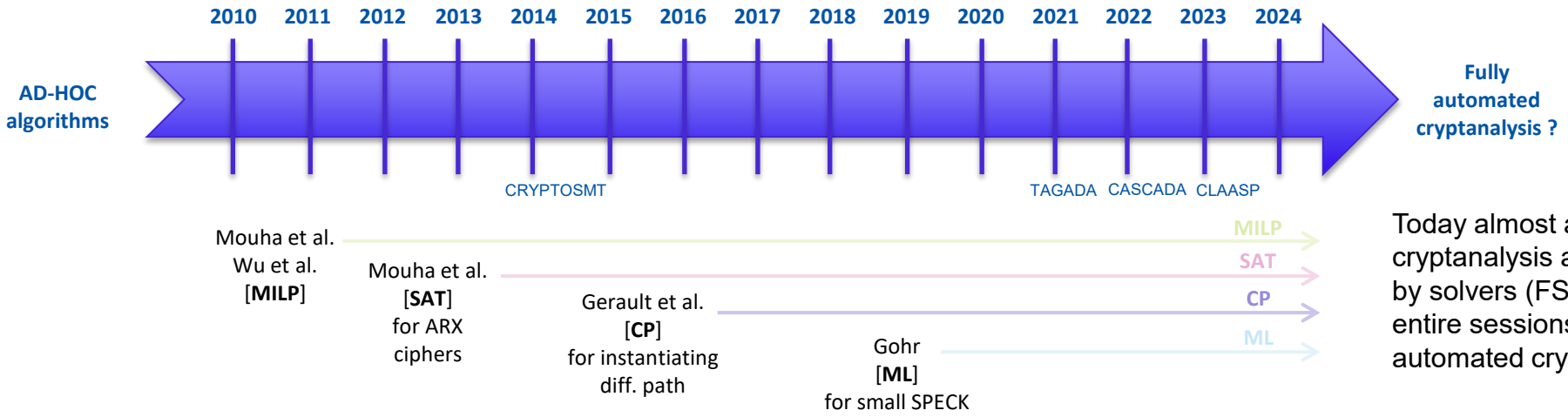
Timeline of Automated Cryptanalysis



Today almost all cryptanalysis are assisted by solvers (FSE 2024: two entire sessions just for pure automated cryptanalysis)

Automated cryptanalysis using declarative frameworks (SAT/MILP/CP/etc.) is generally slower or at best same as ad-hoc tools, but so much **more convenient**

Mainly on **differential** and **linear cryptanalysis**, but now also on integral distinguishers, cube attacks, meet-in-the-middle attacks, etc.

Solving time is a crucial aspect and can be impacted by:

- the framework you use (SAT/MILP/CP/etc.)
- the strategy of modeling
- the solver
- the type of problem studied / scale

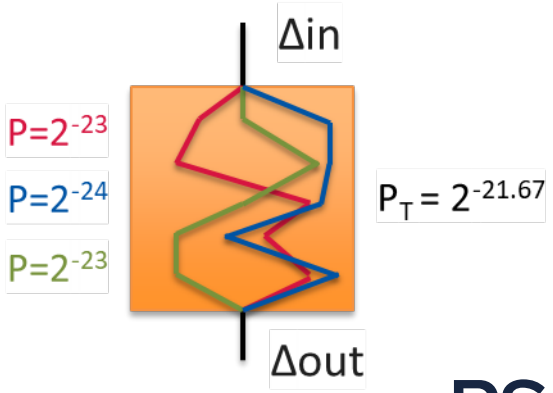
Automated Cryptanalysis for Differential Paths

Typically, for finding **differentials** or **differential trails**:

- Use **variables** to represent the various stages of the internal state bit differences during the round (and throughout the rounds)
- Use other variables to represent the **probability P** of the differential path (in -log2)
- Model a round of the cipher as a set of **declarative constraints** (Markov assumption !) to represent the difference propagation (either truncated or not). Use temporary variables if needed for certain components.
- Put all this into a system and use a **solver** on it.
- Can be combined with extra upper-level strategies (Matsui branch-and-bound, etc.)

One can:

- Find the best differential path / linear characteristic
- Enumerate the number of solutions
- Estimate the probability of a differential



Satisfiability Problem (SAT/SMT) for Cryptanalysis

All **variables** are **Boolean**, a **constraint** is a **Conjunctive Normal Form** (CNF – conjunction of disjunctions/clauses). Ex: $(a \vee b) \wedge (\neg a \vee c \vee d) \wedge (\neg b \vee c)$

- **Equality** is easy to model $x = y \Leftrightarrow (\neg x \vee y) \wedge (x \vee \neg y) = 1$
- **Linear layers** are easy to model, by combining the simple **XOR** model:
  $x = a \oplus b \Leftrightarrow (\neg a \vee b \vee x) \wedge (a \vee \neg b \vee x) \wedge (a \vee b \vee \neg x) \wedge (\neg a \vee \neg b \vee \neg x) = 1$
  (use dummy variables with combinations of XORs for multiple-inputs XORs)
- **Nonlinear layers (Sbox/AND/OR/Additions)** are more complex to model

Then, add a constraint to force the path to have a certain fixed probability P:

- If the solver returns SAT, we directly get a path with probability P.
- If the solver returns UNSAT, we run again this time with a probability P' < P (**drawback:** we need to iterate through decreasing target values).

SAT is **very good for all ciphers,** especially for **ARX ciphers**
SAT is **very good** for **finding optimal differential paths**, and **good** for **estimating the probability of a differential**.

Mixed Integer Linear Programming (MILP) for Cryptanalysis

All **variables** are **Boolean/integer/real**, a **constraint** is a linear **inequality**
Ex: $\{a + 4b + 2c \geq 3d + 7e\}$

- **Equality** is easy to model $x = y \Leftrightarrow \{x \geq y\}, \{y \geq x\}$
- **Linear layers** are easy to model, by using **4 inequalities**:
  $x = a \oplus b \Leftrightarrow \{a + b \geq x\}, \{a + x \geq b\}, \{b + x \geq a\}, \{a + b + x \leq 2\}$
  (use dummy variables with combinations of XORs for multiple-inputs XORs)
- **Nonlinear layers (Sbox/AND/OR/Additions)** are more complex to model

Then, add an **objective function** to minimize/maximize (encoding the path probability). Ex: $\{a + 3b + 2c + 5d\}$. The solver will directly return the best instance identified.

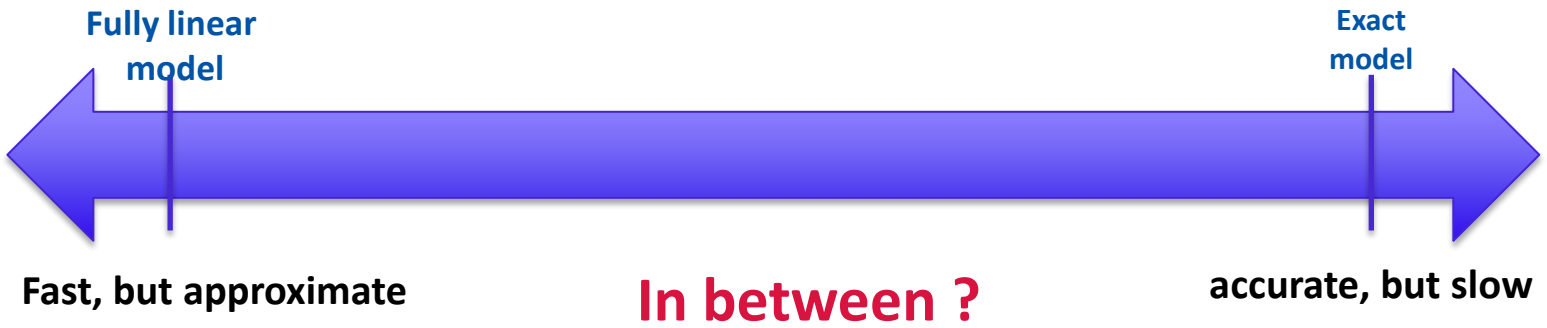
MILP is **good** on **most ciphers**.
MILP is **very good** for **finding optimal differential paths**,
less so for **estimating the probability of a differential**.

The Problem with Modular Addition Modeling

- **Sboxes are relatively easy to model:** transform DDT into constraints
- **You can't use that strategy for ARX:** size is too large (32/64 bits)
- **To overcome this, researchers tried:**
  - to **fully linearise** mod. addition (Ex: MDx/SHA-x collision attacks): $x = a \boxplus b \approx a \oplus b$
  - to use Lipmaa-Moriai formula for XOR-differential propagation through addition:

    for $(\alpha, \beta \rightarrow \gamma)$: $eq(\alpha \ll 1, \beta \ll 1, \gamma \ll 1) \wedge (\alpha \oplus \beta \oplus \gamma \oplus (\beta \ll 1)) = 0$, with $eq(x, y, z) := (\neg x \oplus y) \wedge (\neg x \oplus z)$

  - to build the differential path iteratively, **only propagating the most likely differences** at each round (not representing the full cipher in a single model): partial DDT (pDDT)



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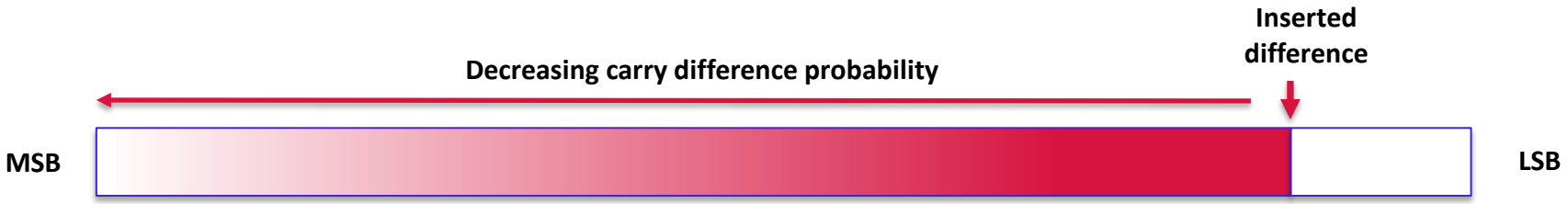
The Window Heuristic

The Windows Heuristic Idea

A difference on bit i of a modular addition input can create a differential carry propagation from i to many later positions.

A linear model forces 0 carry difference propagation (but is fast), while exact model allows any propagation (but is slow).

- **Our observation:** good differential paths are (almost always) composed of modular addition diff. transitions that are propagating very little (if not at all), with only a few rounds with more propagation.



The Windows Heuristic Idea

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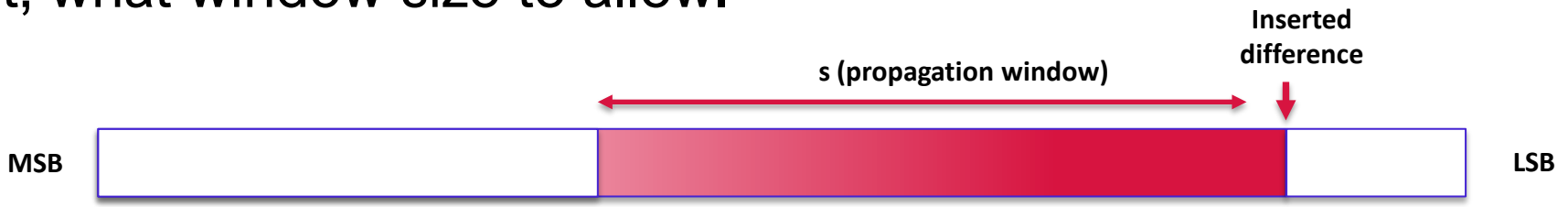
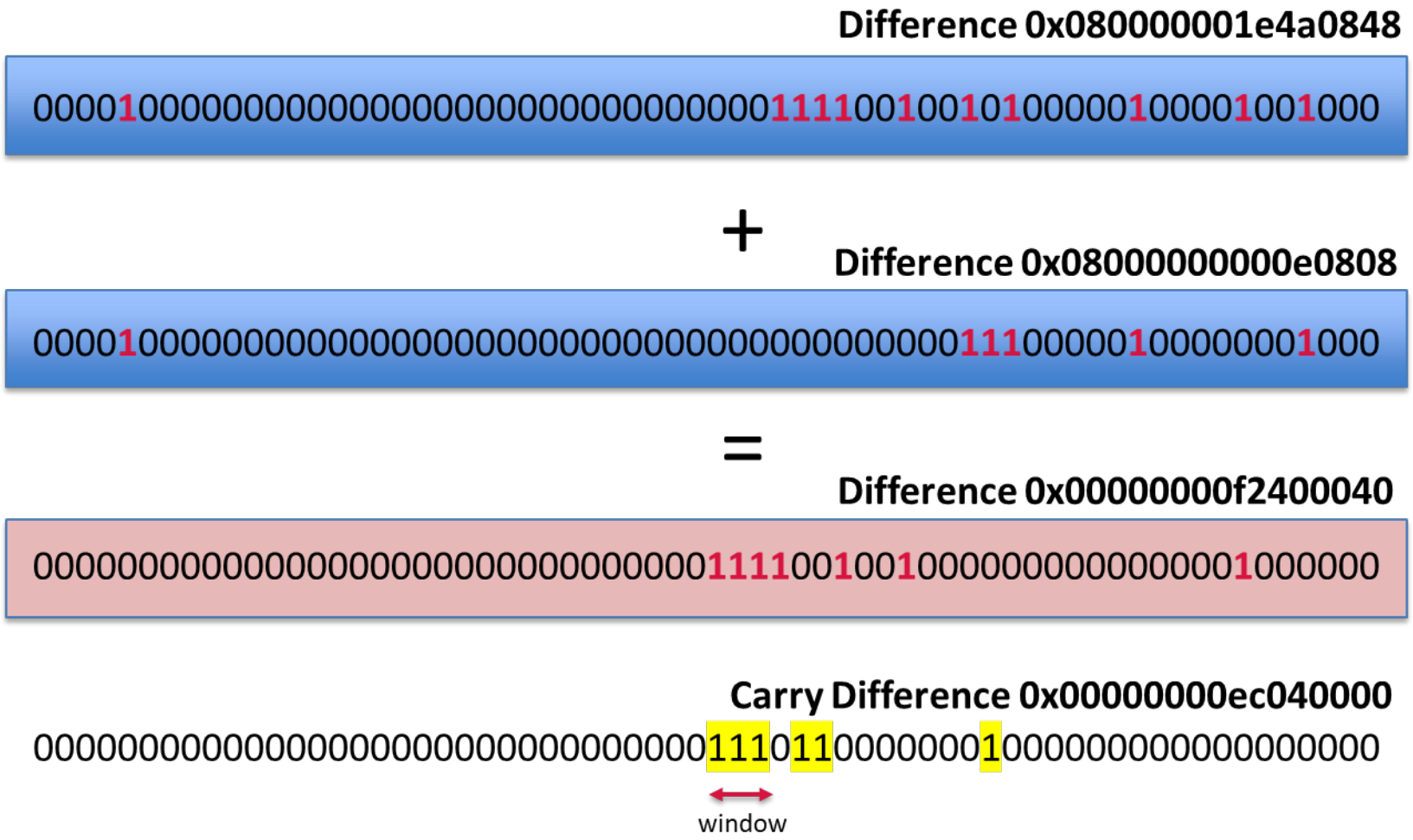
A linear model forces 0 carry difference propagation (but is fast), while exact model allows any propagation (but is slow).

- **Our observation:** good differential paths are (almost always) composed of modular addition diff. transitions that are propagating very little (if not at all), with only a few rounds with more propagation.
- **Our idea:** generalize the linear and exact models, by allowing a difference **in the carry** to propagate a little bit (over s bits, aka the "**window**" of propagation). We can control for each modular addition and each difference bit of it, what window size to allow.



The Windows Heuristic Idea

Example:



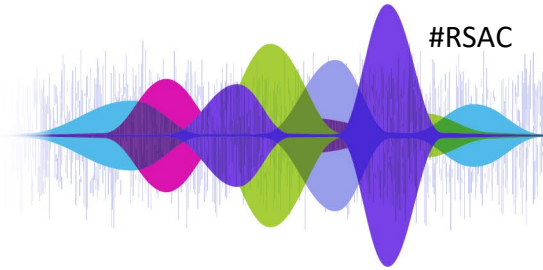
Encoding the Window in MILP and SAT

- In **MILP**, for each n-bit modular addition with s-bit window, we add the (n - 1 - s) following new constraints on the corresponding carry C:

$$\sum_{i=1}^{s+1} C[j-i] \leq s, \qquad for\ all\ j \in [s+1, \ldots, n-1].$$

- In **SAT**, for each n-bit modular addition with s-bit window, we add the $2^{2(s+1)}$ (n - 1 - s) following new constraints on the corresponding carry C:

$$\bigwedge_{j=s+1}^{n-1} \bigvee_{i=1}^{s+1} \neg(a[j-i] \oplus b[j-i] \oplus c[j-i])$$



The Heuristics

Window Heuristic does not create invalid paths, but potentially miss good ones (good to find attacks, bad to prove the non-existence of diff. paths).

We observed that **fixing s for an entire round** generally works well in practice, but this remains dependent on the cipher and attack we study.

Other possible heuristics:

- Force each round to contain few consecutive carry difference clusters (in general small for good differential paths)
- Restrict the **total** number of carry differences for each modular additions (in general not too large for good differential paths)

Unfortunately, these other heuristics did not improve the results.

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Results

Application to SPECK Block Cipher

Version	Rounds	$\log_2(P_{DK})$	$\log_2(P_D)$	$\log_2(P_K)$	Ref.
SPECK-32/64	15	-94.0	−32.0	−62.0	[42]
	15	-85.0	−32.0	−53.0	[41]
	15	-73.0	−31.0	−42.0	$w_s = 0$
SPECK-48/96	14	-68.0	−43.0	−25.0	[42]
	14	-66.6	−43.0	−23.6	[41]
	14	-65.0	−34.0	−31.0	$w_s = 0$
	15	-89.0	−46.0	−43.0	[42]
	15	-83.5	−42.0	−41.5	[41]
	15	-75.0	−41.0	−34.0	$w_s = 0$
	16	-86.0	−43.0	−43.0	$w_s = 0$
SPECK-64/128	14	-88.0	−37.0	−51.0	[42]
	14	-72.0	−35.0	−37.0	[41]
	14	-66.0	−32.0	−34.0	$w_s = 0$
	15	-105.0	−45.0	−60.0	[42]
	15	-89.0	−42.0	−47.0	[41]
	15	-76.0	−33.0	−43.0	$w_s = 0$
	16	-103.0	−60.0	−43.0	[42]
	16	-85.0	−39.0	−46.0	$w_s = 0$
	17	-112.0	−62.0	−50.0	[42]
SPECK-128/256	16	-121.0	−76.0	−45.0	[42]
	16	-96.0	−52.0	−44.0	$w_s = 0$
	19	-190.0	−111.0	−79.0	[42]
	19	-171.0	−102.0	−69.0	$w_s = 0$

[41] Qin et al.
CoRR abs/2203.09741 - 2022

[42] Sadeghi et al.
Des. Codes Cryptography 2021

Application to ChaCha

Differential paths on ChaCha - comparison

Number of rounds	MILP + window heur.		SAT + window heuristic + "full window size"	MILP [7]	S-function [7]
	Pr	w_s	Pr	Pr	Pr
1	3	3	3	3	3
2	37	3	37	37	37
3	120	1	120	147	157
4	218	1	217	316	349

[7] Bellini et al. - IJACT 2023

Differential paths on ChaCha with MILP

r	Single-key			
	no-condition		conditions	
	Time	Pr	Time	Pr
1	2.17s	3	1.62s	3
2	13.2s	37	3.30s	37(0)
3	13.6s	147	319s	120(0)
4	36728s	316	48745s	218(1)

Boomerang Distinguishers on ChaCha

Number of rounds	Theoretical probability	Experimental probability
2	2^{-6} ($w_s = -1$, bottom part starting round 2)	$2^{-0.05}$
3	2^{-10} ($w_s = -1$, bottom part starting round 2)	$2^{-1.41}$
4	2^{-90} ($w_s = 0$, bottom part starting round 3)	-
5	2^{-154} ($w_s = 1$, bottom part starting round 3)	-
6	2^{-228} ($w_s = 1$, bottom part starting round 3)	-

Application to LEA Block Cipher

Differential paths on LEA

r	Window size		Others			
	Pr	w_s	Pr	Ref.	Pr	Ref.
12	107	9	112	[43]	107	[3]
13	123	1	134	[43]	127	[3]

Differential paths on LEA (time)

r	Single-key			
	no-condition		conditions	
	Time	Pr	Time	Pr
12	100709s	107/(-1)	73405s	107/(9)
13	609.53s	143/(-1)	7985s	123/(1)

[3] Bagherzadeh et al. - IET Information Sec. 2020

[43] Song et al. - ACISP 2016

Key recovery on LEA

Distinguisher			Key recovery		
Rounds	Pr	Ref.	Rounds	Complexity	Ref.
13	$2^{-123.79}$	[43]	14	$2^{124.79}$	[43]
13	$2^{-110.97}$	Ours ($w_s = 1$)	14	$2^{111.97}$	Ours ($w_s = 1$)

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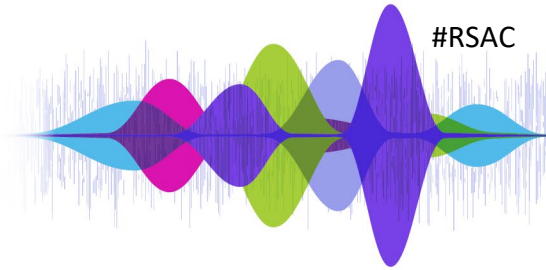
Conclusion
-
Future Works



Conclusion

- Maybe **better heuristics** exist with such window modeling trick
- Generalize the way we limit the carry, **maybe other good representations** are possible
- Generalize to **more complex operations** cases (multiplications, algebraic formula, ..)
- What about **linear cryptanalysis** ? Truncated differential cryptanalysis ?
- Incorporate the Window Heuristic into larger models with **key recovery**

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SESSION ID: CRYP-M01B

Quasidifferential Saves Infeasible Differential: Improved Weak-Key Key-Recovery Attacks on Round-Reduced GIFT

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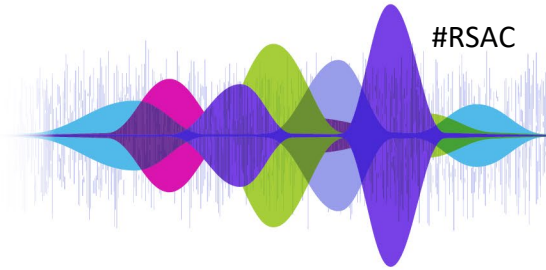

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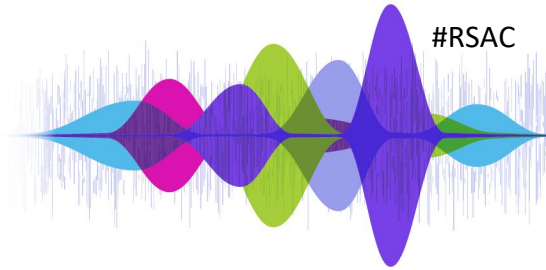

Overview

- Objective:
  - Enhance differential attacks on GIFT using quasidifferentials under weak-key conditions.
- Contributions:
  - Amplified differential probabilities in the weak-key space.
  - Improved weak-key key-recovery attacks on GIFT-64 and GIFT-128.
  - Reveal vulnerabilities in GIFT's weak-key space and inform future lightweight cipher design.

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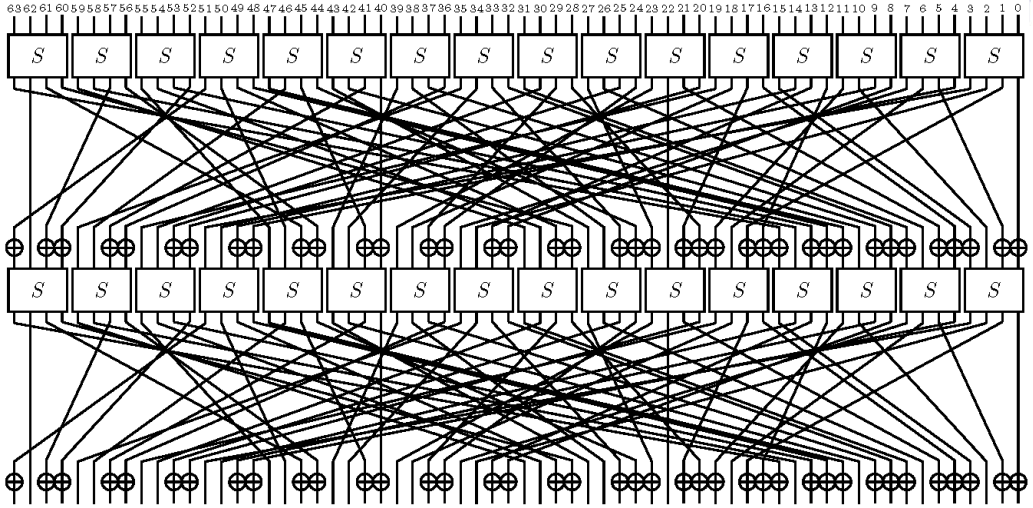
Background & Motivation

GIFT

SPN structure

- Round function:
  - SubCells.
  - PermBits.
  - AddRoundKey.
- Two variants:
  - GIFT-64 (28 rounds).
  - GIFT-128 (40 rounds).



Limitations of Prior Work

- Best attacks:
  - 21 rounds (GIFT-64).
  - 27 rounds (GIFT-128).

Question

- Challenges:
  - Low-probability differentials limit attack feasibility.
  - Existing methods ignore key-dependent probability amplification.
- Key Question:
  - “Can weak-key conditions amplify infeasible differentials?”
- Key Idea:
  - Quasidifferential cryptanalysis (Beyne & Rijmen, CRYPTO 2022).

Methodology

- Step 1: Weak-Key Extraction.
  - Derive linear equations from the quasidifferential trails.
  - Convert round-key bits to master-key conditions.
- Step 2: Probability Amplification of Distinguishers.
  - Sum trails with |correlation| = p_{avg}.
- Step 3: Key Recovery.
  - Extend distinguishers with additional rounds and launch attacks.

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Weak-Key Distinguishers

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Deduce Weak-Key Conditions

- Compute probability:
  - Fixed-key probability = sum of quasidifferential trail correlations.
  - One trail's correlation is $cor = \prod_{i=1}^{r}(-1)^{u_{i+1}^T k_i} D^{G_i}_{(u_{i+1},a_{i+1}),(u_i,a_i)}$
  - Example:

	u in one trail	a in one trail	D^{G_1}	$u_2^T k_1$
round 1	u_1: 0x0000000000000000	a_1: 0x0000000000000202	2^{-4}	$rk_1^0+rk_1^{16}$
	u_2: 0x0000000500000005	a_2: 0x0000000500000005		

ROUND KEY $rk_1^0 + rk_1^{16} \rightarrow$ **MASTER KEY:** $k_0^0 + k_0^8$.

The probability of round 1 is $((-1)^{k_0^0+k_0^8}+1) \times 2^{-4} = \begin{cases} 2^{-3}, k_0^0 + k_0^8 = 0 \\ 0, \quad k_0^0 + k_0^8 = 1 \end{cases}$.

Weak-Key Distinguishers on GIFT-64

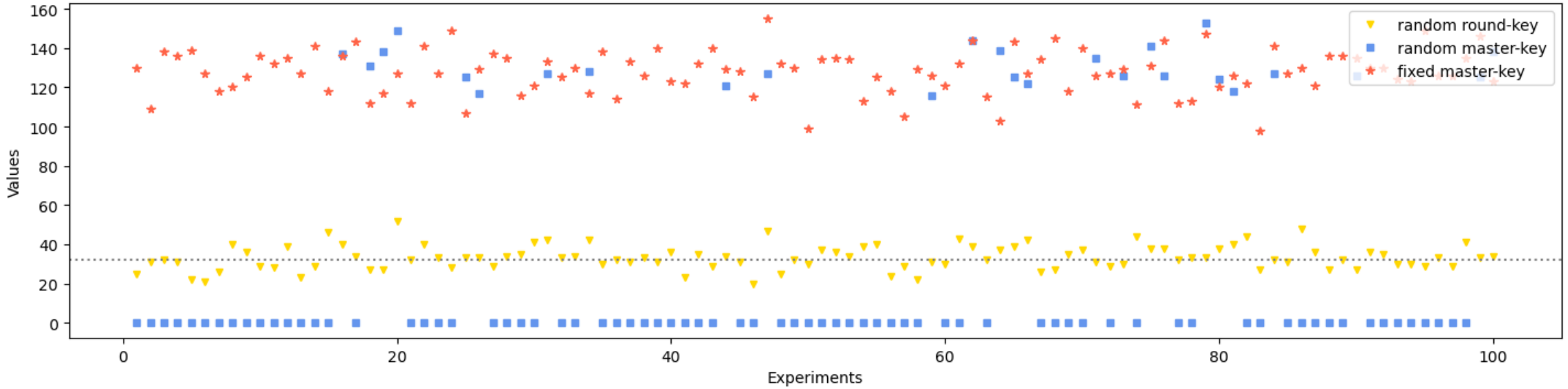
13-round differential:

- Weak-key conditions:
  - $k_0^4+k_0^{12} = 0$
  - $k_4^5+k_4^{13} = 0$
  - $k_0^0+k_0^8 = 0$
  - $k_4^1+k_4^9 = 0$
- Probability:
  - $2^{-62.06} \rightarrow 2^{-57.82}$

13-round multiple differentials:

- Weak-key conditions:
  - $k_2^1+k_2^9 = 0$
  - $k_2^5+k_2^{13} = 0$
  - $k_6^0+k_6^8 = 0$
  - $k_6^4+k_6^{12} = 0$
- Probability:
  - $2^{-58.96} \rightarrow 2^{-55.67}$

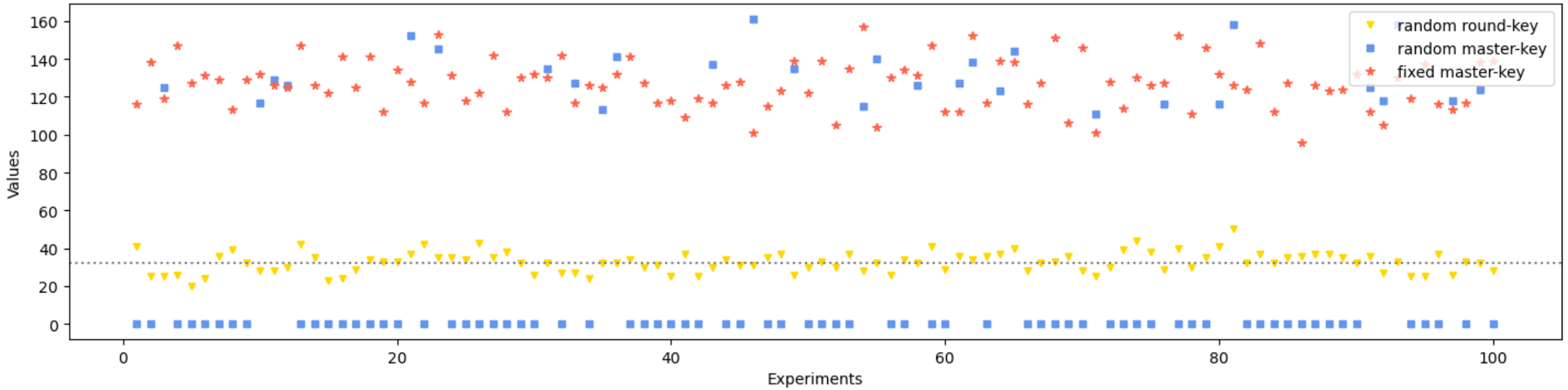
Experiments on GIFT-64



ROUND 1 TO 4 OF 13-ROUND DOMINANT CHARACTERISTIC OF GIFT-64.

Probability of round 1 to 4: $2^{-18.00}$ (4-bit weak-key conditions) vs 2^{-20} (average).

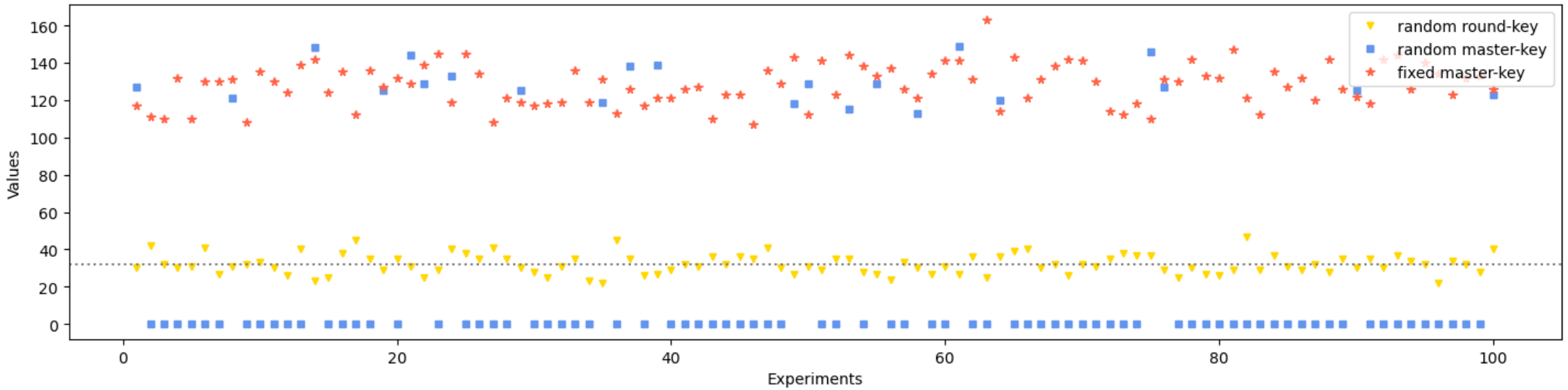
Experiments on GIFT-64



ROUND 5 TO 8 OF 13-ROUND DOMINANT CHARACTERISTIC OF GIFT-64.

Probability of round 5 to 8: $2^{-18.01}$ (4-bit weak-key conditions) vs 2^{-20} (average).

Experiments on GIFT-64



ROUND 9 TO 13 OF 13-ROUND DOMINANT CHARACTERISTIC OF GIFT-64.

Probability of round 9 to 13: $2^{-22.00}$ (4-bit weak-key conditions) vs 2^{-24} (average).

Weak-Key Distinguishers on GIFT-128

20-round differential:

- Weak-key conditions:
  - $k_0^9 + k_1^7 = 0$
  - $k_0^{11} + k_1^9 = 0$
  - $k_4^5 + k_5^1 = 0$
  - $k_6^9 + k_7^{11} = 0$
  - $k_6^{11} + k_7^{13} = 0$
  - $k_2^{15} = 1$
- Probability:
  - $2^{-121.83} \rightarrow 2^{-114.77}$

21-round multiple differential:

- Weak-key conditions:
  - $k_4^4 + k_5^0 = 0$
  - $k_6^8 + k_7^{10} = 0$
  - $k_6^{10} + k_7^{12} = 0$
  - $k_2^1 = 0$
- Probability:
  - $2^{-128.38} \rightarrow 2^{-122.77}$

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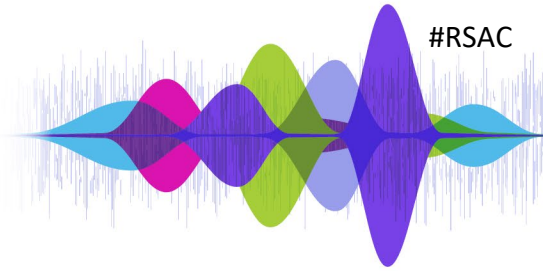
Improved Weak-Key Key-Recovery Attacks

Key-Recovery Attacks on GIFT-64



21-round differential attack:

- Complexity comparison:
  - Time: $2^{120.60}$ (ours) vs. $2^{121.66}$
  - Data: 2^{64} (ours) vs. 2^{64}
  - Memory: 2^{96} (ours) vs. 2^{96}
  - P_S: 99.41% (ours) vs. 51.06%

21-round multiple differential attack:

- Rounds: 21-round (first)
- Complexity comparison:
  - Time: $2^{123.27}$
  - Data: 2^{64}
  - Memory: 2^{112}
  - P_S: 99.9%

Key-Recovery Attacks on GIFT-128

27-round differential attack:

- Complexity comparison:
  - Time: $2^{115.77}$ (ours) vs. $2^{124.83}$
  - Data: $2^{115.77}$ (ours) vs. $2^{123.53}$
  - Memory: 2^{92} (ours) vs. 2^{80}
  - P_S: 99.41% (ours)

28-round multiple differential attack:

- Rounds: 28-round (first)
- Complexity comparison:
  - Time: $2^{123.77}$
  - Data: $2^{123.77}$
  - Memory: 2^{96}
  - P_S: 86.5%

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If you have any questions, please contact chengcheng.chang@mail.sdu.edu.cn.